Hypertext Structured Proof Practice: Induction of Divisibility

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February 2020

Induction of Divisibility

Use induction to prove that $4|(7^n - 3^n)\ \forall n \geq 0$
Proof:
Let n=0, Then P(0)= $7^0 - 3^0 = 0$
Therefore, 4|0 and $4|(7^n - 3^n)$

Assume P(k) is true $\forall k \geq 0$ such that $4|(7^k - 3^k)$ is true.
Show that P(k+1) is also true. That is $\exists r \epsilon Z : 7^{k+1} - 3^{k+1} = 4r$.
$4|7^k - 3^k$ symbolically $\exists l \epsilon Z : 7^k - 3^k = 4l$
$7^{k+1} - 3^{k+1} = 7^k \cdot 7 - 3^k \cdot$
$= 7(4l + 3^k) - 3^k \cdot 3$
=$4(7l + 3^k)$
Let $7l + 3^k = r \epsilon Z$
Then $4(7l + 3^k) = 4r$
Therefore, $4|7^{k+1} - 3^{k+1}$
Proven by the mathematical induction principle.

Explanation of Divisibility

Definition of Divisibility: If n and d are integers then n is divisible by d if, and only if, n equals d times some integer and d is not equal to zero.

Therefore 4 divides 0, zero times: 0=4*k when k=0

Substitution Breakdown

From the Induction hypothesis we have:$7^k - 3^k = 4l$

By rearranging this statement we"ll get: $7^k = 4l + 3^k$

Now expanding the statement we want to prove $7^{k+1} - 3^{k+1} = 7^k \cdot 7^1 - 3^k \cdot 3$

We can now substitute to get: $7(\text{4l+}3^k) - 3^k \cdot 3$

$28l + 7 \cdot 3^k * 3 - 3^k \cdot 3$

$28l + (7-3)3^k$

$28l + 4 \cdot 3^k$

$4(7l + 3^k)$

Let $7l + 3^k = r \epsilon Z$

Then $4(7l + 3^k) = 4r$

Therefore, $4|7^{k+1} - 3^{k+1}$