Chapter 3.1: Limit of Functions Homework

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Exercise 3.1.1 Find the limit or prove that the limit does not exist: Limit$\left[\sqrt{x}, x \to c\right]$

let Limit$\left[\sqrt{x}, x \to c\right] = \sqrt{c}$ for c ≥ 0

now, make $|\sqrt{x} - \sqrt{c}|$ arbitrarily small by making |x-c| small enough

(think of this as a difference of two squares):

let $x = (\sqrt{x})$^2 and c=$(\sqrt{c})$^2

therefore |x-c|= $|\sqrt{x} - \sqrt{c}|\,|\sqrt{x} - \sqrt{c}|$

$=|\sqrt{x} - \sqrt{c}| = \frac{|x-c|}{\sqrt{x}+\sqrt{c}}$

Choosing a random interval from c to $c + \frac{3c}{4}$ and c to $c - \frac{3c}{4}$

take $\frac{c}{4} \leq x \leq \frac{7c}{4}$

then $\frac{-3c}{4} \leq x - c \leq \frac{3c}{4}$

so |x-c|=$\frac{3c}{4}$

and $\sqrt{x} \geq \frac{\sqrt{5}}{2}$

so $\sqrt{x} + \sqrt{c} \geq \frac{\sqrt{c}}{2} + \sqrt{c} = \frac{3\sqrt{c}}{2}$

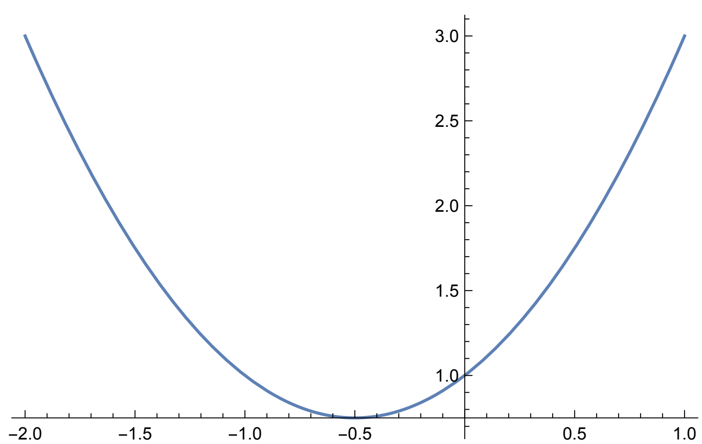
and $|\sqrt{x} - \sqrt{c}| \leq \frac{2}{3\sqrt{c}} |x-c|$

yes, the limit exists

Limit$[x^2 + x + 1, x \to c]$ for any c∈ℝ

In[•]:=
```
Plot[x^2 + x + 1, {x, -2, 1}]
```

Out[•]=



$|f(x)-L| = |(x^2+x+1)-(c^2+c+1)|$

$=|(x^2+x)-(c^2+c)|$

$=|x-c||x+c+1|$

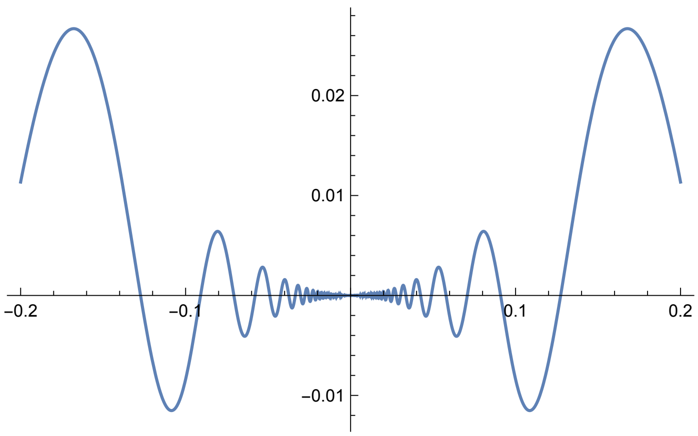
|x-c|=|x-1|<δ

|f(x)-L| < δ(|x|+|c|+1)
then because 0<|x-c| <1 and |X| < |c|+1

|f(x)-L| < δ||c|+1+|c|+1| < δ|2|c|+2|
$\delta = \min\left\{1, \frac{\epsilon}{2|c|+2}\right\}$
|f(x)- L| =|(x^2+x+1)-(c^2+c+1)| < $\frac{\epsilon}{2|c|+2}$(|2|c|+2|) = ϵ
|f(x)- L| < ϵ

Limit[x^2 cos(1/x), $x \to 0$]
When x is positive or negative and is approaching zero cos(1/x) will oscillate between one and negative one, so the this limit primarily focuses on the function x^2

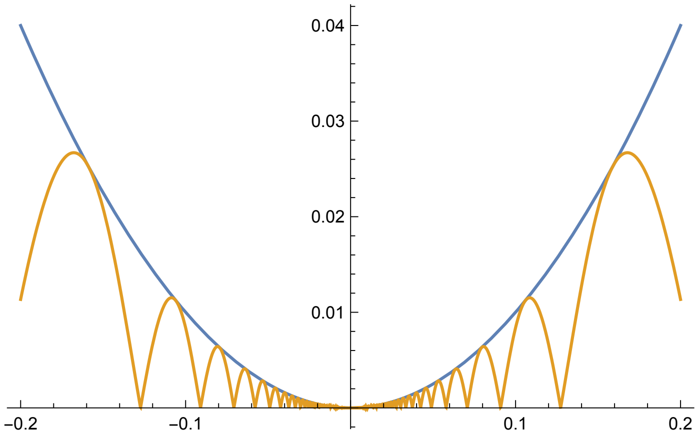
In[ ]:=
```
Plot[x^2 * Cos[1 / x], {x, -0.2, 0.2}]
```

Out[ ]=


The plot above is showing x^2*Cos[1/x] .
Where as the one below is plotting x^2 in blue and the absolute value of cos[1/x] in yellow. This is to show that cos[1/x] < x^2 therefore since cos[1/x] oscillates between 1 and -1 the shape of the graph is dependant on x^2 .

In[∘]:=
```
Plot[{x^2, Abs[x^2 * Cos[1 / x]]}, {x, -0.2, 0.2}]
```

Out[∘]=



exploratory:

|f(x) -L | < ϵ

|x^2 *cos[1/x]-0| < ϵ

|x^2||cos[1/x]| < ϵ

Since the limit of cos[1/x] does not exist we are left with x^2 < ϵ

x< $\sqrt{\epsilon}$

$\delta=\sqrt{\epsilon}$

Proof:

given $\epsilon > 0$

let $\delta=\sqrt{\epsilon}$

then if |x-0| = |x^2 *cos[1/x] -0 |

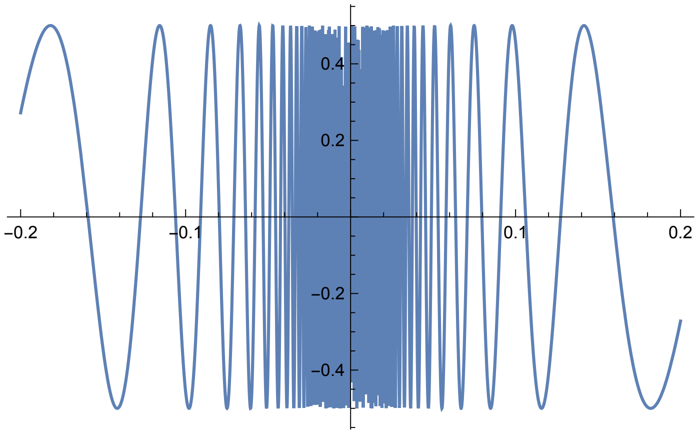
=|x^2 *cos[1/x]| ≤ $|x|^2$

≤ $(\sqrt{\epsilon})^2 \leq \epsilon$

Limit[sin(1/x) cos(1/x), $x \to 0$]

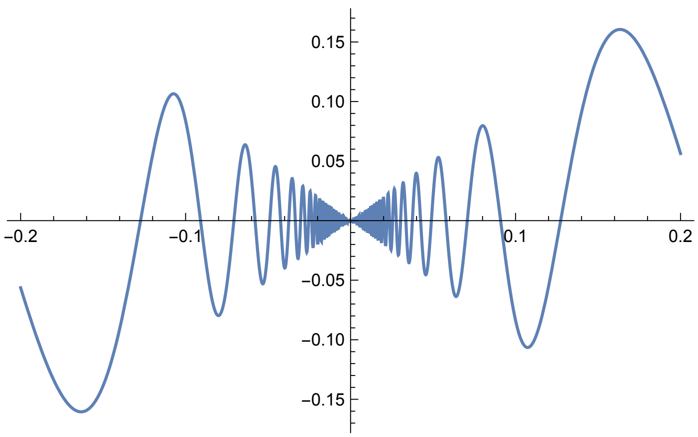
This limit does not exist, as shown in the graph below it oscillates between 1 and -1 for both sin(1/x) and cos(1/x).

In[∘]:= `Plot[Sin[1 / x] * Cos[1 / x], {x, -0.2, 0.2}]`

Out[∘]=


Limit[sin(x) cos(1/x), $x \to 0$]

In[1]:= `Plot[Sin[x] * Cos[1 / x], {x, -0.2, 0.2}]`

Out[1]=


This function approaches zero as x approaches zero, therefore, a limit does exist. Approaching from the negative or positive side of 0, it will eventually lead to zero. (as we are depending on the function sin(x) here and recall that sin(0) is zero.

Example 3.1.8 Find example functions f and g such that the limit of neither f (x) nor g(x) exists as x → 0, but such that the limit of f(x)+g(x) exists as x → 0.

This can be solved by a piecewise function:

let f(x)= $\begin{cases} 5 & \text{if } x \geq 0 \\ -5 & \text{if } x < 0 \end{cases}$

Now the Limit[$f(x)$, $x \to 0$] when $x \geq 0$ approaches 5 and when $x < 0$ it approaches -5

g(x) = $\begin{cases} x-5 & \text{if } x \geq 0 \\ x+5 & \text{if } x < 0 \end{cases}$

Limit[$g(x)$, $x \to 0$] when $x \geq 0$ approaches -5 and when $x < 0$ it approaches 5 f(x)+g(x)= $\begin{cases} 5+(x-5) & \text{if } x \geq 0 \\ -5+(x+5) & \text{if } x<0 \end{cases}$

which simplifies to: $\begin{cases} x & \text{if } x \geq 0 \\ x & \text{if } x<0 \end{cases}$

therefore this limit does not exist